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J. H. Alday  
*Ingersoll-Rand Company*

J. A. Hood  
*Ingersoll-Rand Company*

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# A PROGRAM FOR MATHEMATICAL MODELING AND ANALYSIS OF ROTARY SCREW COMPRESSOR PERFORMANCE

J. H. Alday

J. A. Hood

Development Engineering Department  
Rotary-Reciprocating Compressor Division  
Ingersoll-Rand Company

## ABSTRACT

A computer program has been developed which provides a quick, simple, and accurate environment for modeling and analyzing the performance of a rotary screw compressor. The program utilizes performance data measured during dynamometer testing of a compressor to develop a mathematical model of the compressor's performance. This model is fundamentally different from traditional methods in that this model is a function of multiple performance parameters (discharge pressure, rotor speed), and the equations comprising the model have been developed such that the results are more consistent with accepted performance characteristics. The mathematical model determined by this program provides a single data base for performance, optimization, and statistical analysis that can be used for compressor development and ultimate package installation.

## INTRODUCTION

One of the critical steps in advancing the state of the art in compressor design is evaluating and processing experimental test results. While increasingly sophisticated and accurate data acquisition systems are becoming available, test results are often still analyzed and processed using archaic methods. These "tried and true" methods can be useful when applied with experience and good judgement, but several developments are taking place which emphasize the need for a more advanced method for modeling compressor performance.

1. Increased emphasis on computer aided engineering requires performance models to be mathematically rather than graphically based.
2. The drive toward greater efficiency and lower cost requires the design engineer to utilize both analytical and experimental tools to optimize compressor design.
3. Product development schedules are being compressed. Test results must be timely, accurate, and useful to others, particularly package and application engineers.

This paper will review current performance modeling techniques, introduce criteria for the successful performance modeling program, and describe a program that meets these criteria.

## BACKGROUND

The oldest and simplest method of developing a performance model is hand-fitting curves to experimental test results. Commonly, a separate variable speed curve will be drawn for each pressure tested. The result will be a set of curves for capacity, power, volumetric efficiency, adiabatic efficiency, etc. versus speed for each pressure tested, or alternatively versus pressure for each speed tested.

Although hand fitting curves takes relatively little "up front" effort, this method has several drawbacks which limit its usefulness.

1. The hand-fit curve is very subjective. The degree to which it accurately models the actual performance depends upon the experience and judgement of the person drawing the curve.
2. Data must be interpolated to obtain performance at "off-speed" or "off-pressure" points (points not tested on the dynamometer). Each manual interpolation introduces more error into the process.
3. There is no consistency between different plots of the same data. For any given set of operating conditions, the performance derived from a variable speed plot will not necessarily agree with the performance from a variable pressure plot.
4. This method is not easily integrated into computer-based analytical techniques.
5. This method is not conducive to statistical analysis. Determining the effect of the test error on the test results, for example, depends upon the experience and judgement of the engineer performing the analysis.

The personal computer and its associated graphics and plotting programs have enabled the replacement of the hand-fit method with various mathematical regression analyses. These routines often employ the least squares method and will usually perform only a two variable (one independent, one dependent) curve fit. The two variable least squares (TVLS) curve fit method has two significant advantages over the hand-fit method.

1. Because the fit is mathematically determined, each set of data is fit in the same manner and its accuracy is not dependent upon the judgement of the person performing the analysis.
2. This method is more easily integrated into computer-based systems.

Although the TVLS curve fit is a step forward, it too has several limitations.

1. The equations to which the data are fit are arbitrary and are not related to the true performance characteristics of a rotary compressor. As a result, indicators that are important in analyzing compressor performance, such as slope and curvature of constant speed and constant pressure plots, are often incorrect.
2. A different equation is developed for each parameter of the test. Interpolating and extrapolating performance is as uncertain as it is with the hand-fit method.
3. Consistency is not guaranteed between variable speed and variable pressure plots of the same data. This is due to having an independent equation for every speed and every pressure run in the compressor test.
4. Because each curve is derived on the basis of only a few measured points and is independent of the other curves, the benefits of statistical techniques for evaluating the confidence of the resulting performance model are limited.

There are more sophisticated programs available that can fit more than a single

independent variable at a time. These programs are intended for general purpose use, however, and still contain deficiencies in terms of their ability to accurately model rotary screw air compressor performance. A study which reviewed the limitations of current methods for modeling compressor performance from experimental data concluded that the successful program would have the following characteristics.

1. Performance over the entire range of pressure and speed should be modeled by a single equation for power as a function of pressure and speed and a single equation for capacity as a function of pressure and speed (each equation would have two independent variables). In this way, there would be no discrepancy in performance due to interpolating between different independent models, and data plotted versus speed would be consistent with data plotted versus pressure.
2. Only measured data (capacity and power) should be fitted. Derived quantities which utilize both power and capacity in their calculation (specific power, adiabatic efficiency) would be calculated using fitted values of power and capacity. This would be essential to assure correlation between measured and calculated results.
3. The program should use statistical means for identifying points that vary significantly from the pattern established by the remaining data. This would minimize the likelihood that the resulting performance model would be skewed by test errors.
4. The mathematical model should be based on the true performance characteristics of a rotary screw compressor. This would increase confidence in the model, especially when using the model for prediction purposes.

These criteria are not limited to the rotary screw, and may be applied to develop a performance model for any type of compressor. The main difference that would exist between models of different compressor types would be the equations comprising the model itself. These equations must reflect the performance characteristics of the particular type of compressor being modeled.

In response to any arguments that a performance modeling program is not needed if dynamometer data is of sufficient quality, the following points should be considered.

1. Regardless of the quality of instrumentation used in performance testing, there will always be random test error in the data. A good mathematical model can help to compensate for this error.
2. A mathematical model can make performance analysis more efficient by decreasing the amount of data required in each dynamometer test of a compressor and by decreasing the time required to get results to the engineer.

## PROGRAM LOGIC AND DEVELOPMENT

The Airend Performance Data Analysis Program (APDAP) was developed in response to the above criteria. Upon execution, APDAP reads power, capacity, discharge pressure, and male rotor speed data measured during the dynamometer test of the particular compressor to be modeled. These data have been corrected to a standard inlet pressure, but otherwise they are "raw" measured data. From these data, the program generates a data base of pertinent quantities, such as specific power and volumetric and adiabatic efficiency, to be used for plotting and comparison to the model determined by the program. Next, the program performs a least squares fit to the dynamometer data by using matrix algebra to calculate the coefficients of the two

equations comprising the mathematical model. The program forms a  $n \times n$  matrix ( $n$  = the number of dynamometer data points read by the program) from the dynamometer data based on the form of the model equations. To estimate the accuracy of the coefficients that would be calculated without actually solving the matrix, the condition number of the matrix is estimated. If the condition number exceeds  $10^{17}$  then the coefficients would have no accuracy and the program is terminated without solution. This usually only occurs with excessively small sets of dynamometer data, and is discussed later in this section. Under normal circumstances, the matrix and a vector of measured power are passed to a subroutine which solves the matrix equation. The matrix solution yields the coefficients of the power equation so that it best fits the dynamometer data. This process is then repeated to determine the coefficients for the capacity equation. Now a pair of equations exist which completely model the compressor performance (power as a function of discharge pressure and rotor speed, capacity as a function of discharge pressure and rotor speed) over the complete range of conditions tested on the dynamometer. These equations are used to generate a data base of "fit" quantities for comparison to those calculated directly from the dynamometer data. The last module of the program allows the user to create data files from the dynamometer and mathematical model data bases for plotting purposes.

A major portion of the program development involved finding the best form of equations to use in the mathematical model. Initially, the effort was to find an equation form that best fit the dynamometer data. In other words, the difference between the value predicted by the model and that measured in the lab was to be minimized. However, this reduced the model to nothing more than a curve fitting routine, and caused any experimental error from the lab to be manifested in the model equations. As mentioned earlier in the criteria for the successful program, the objective is to have equations which predict the true compressor performance based on measured data. Testing experience and analytical studies suggest what graphs of certain performance quantities versus discharge pressure and rotor speed should look like. The trick is to find equations that model these known trends and also fit actual test data in an accurate fashion. As an example, the first equation attempted fit the data well, yielding a low root mean square difference between predicted and measured performance. However, the first derivatives of the power versus speed curves derived from the model were the same for every pressure at each speed. In other words, the power versus speed curve for a discharge pressure of 100 psig was parallel to the curve for 180 psig. This trend was evident in all performance data plotted from the model. Even though these curves fit the data fairly well, it was obvious that they did not correctly model the true physics of the compressor performance. Once equations were found that corrected this problem with the first derivative, a similar exercise was performed to optimize the behavior of the second derivative of the curves derived from the model.

After best equation form for the model had been determined, the effects of dynamometer data set size and compressor size were studied. As the number of points in the data set was reduced, the difference between the fit curves and the measured data decreased, but the model no longer accurately reflected trends of the compressor physics. Recall that the purpose of the model is to interpret dynamometer data rather than to simply match it. Additionally, in data sets with fewer than 15 points (five speeds and three pressures) the condition number of the matrix to be solved tended to increase, indicating the accuracy of the coefficients calculated for the model was decreasing. While basing the model on too few measured points reduced confidence in the model, requiring an excessive amount of data negated any benefits derived from the model due

to the increased cost of compressor testing. Optimum data set size was found to be from 15 to 25 points, with a minimum of three pressures or speeds in each set. When the effects of data set size and equation form were studied on different size compressors, identical trends in the model results were seen, indicating that the same equation form is valid for both large and small compressors.

One criterion for the successful program mentioned in the last section was that statistical means should be used to identify data points that vary significantly from the rest of the data set. This is important because even though APDAP is less sensitive to experimental error than other methods, certain types of poor data points, especially those high in capacity and low in shaft power, or vice versa, and at an extreme speed and pressure, may effect the model substantially. As a result, after performance data is calculated from the "raw" dynamometer data read by the program, this data base is scanned for inconsistencies in trends of power, capacity, and adiabatic and volumetric efficiency. If a data point is found that appears to be inconsistent with the other data, the user is given the option of discarding that point for purposes of developing the model for that data set. Results after discarding points such as these are greatly improved. This reiterates the requirement of good judgement to produce accurate performance curves from measured data. Blindly passing curves through the dynamometer data does not ensure a model that accurately represents the compressor performance.

## RESULTS

Once the performance modeling program was written, it was necessary to evaluate its validity. This was challenging because the "real" (independent of test error) performance is not available. Conventional correlation techniques can only evaluate how well the model correlates to measured data. What is needed is a way to filter out the effect of routine measurement inaccuracy to evaluate how well the program models the real compressor performance.

In order to more thoroughly assess the accuracy of the APDAP program, the following procedure was developed as part of the evaluation process.

1. A "theoretical" model was developed from the results of many tests of several compressors. This model is representative of what is judged to be the real performance of a typical rotary screw compressor.
2. A set of simulated test points was developed which reflects the results of a typical performance test of the "theoretical" compressor. Although these points are arbitrary in nature, care was taken to ensure they possess typical random error inherent to performance testing.
3. The results of this simulated test were modeled using both the TVLS curve fit method and the APDAP program. These models were evaluated not only in terms of how well they correlated to the data points, but also in terms of how accurately the theoretical performance characteristics were represented.

Figure 1 shows how well the two models correlate to the test points. As can be seen from the values of root mean square difference presented in Table I, there is little difference between the two methods from the standpoint of conventional statistical analysis. Both methods match the simulated experimental data reasonably well and either method would probably be considered acceptable.

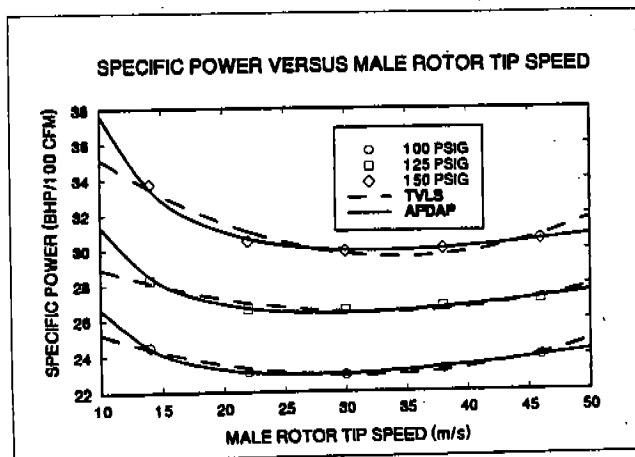
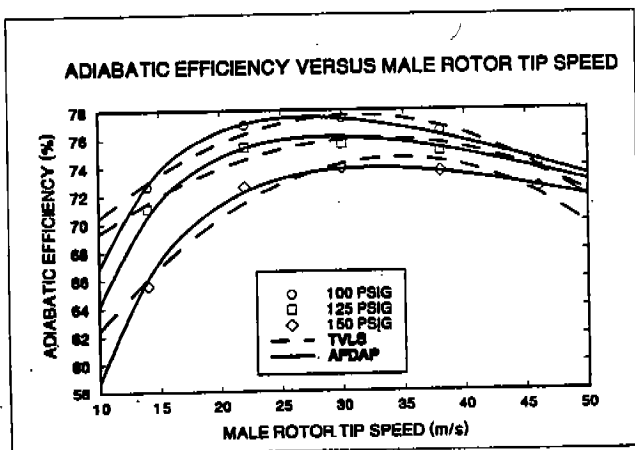
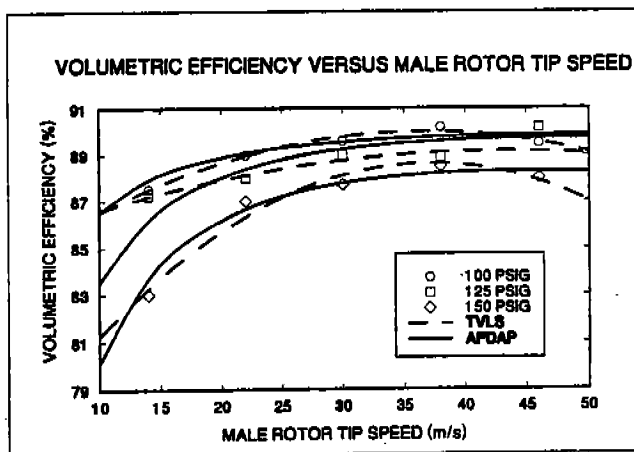


Figure 1. Comparison of APDAP and TVLS to Simulated Data Points

**Table I. Root Mean Square Difference from Simulated Data Points**

PERFORMANCE VARIABLE	TVLS RESULTS	APDAP RESULTS
ADIABATIC EFFICIENCY (%)	0.548	0.280
VOLUMETRIC EFFICIENCY (%)	0.370	0.511
SPECIFIC POWER (BHP/100 CFM)	0.237	0.115

Figure 2 shows how the two models correlate to the theoretical performance characteristics from which the test points were derived. After studying the graphs in this figure, two differences between the APDAP and TVLS models become apparent.

1. The APDAP model more accurately indicates optimum speed. At 150 psig for example, the TVLS curves suggest an optimum speed near 35 m/s for both volumetric and adiabatic efficiency. In contrast, the APDAP curves indicate a lower optimum speed that more accurately agrees with the theoretical performance.
2. The slopes of the APDAP curves more accurately match the slopes of the theoretical curves. This is especially apparent at low speeds.

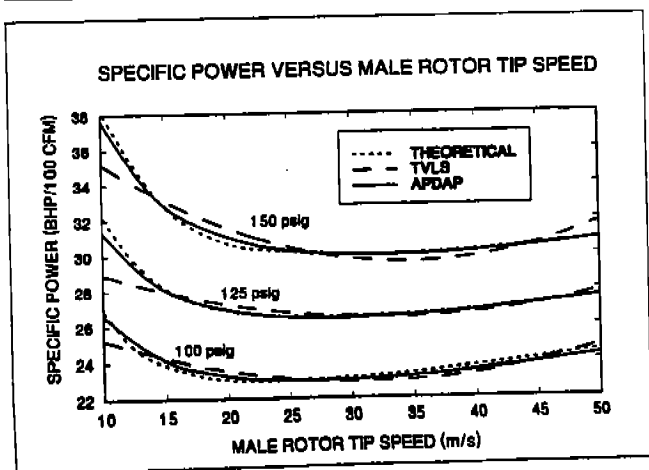
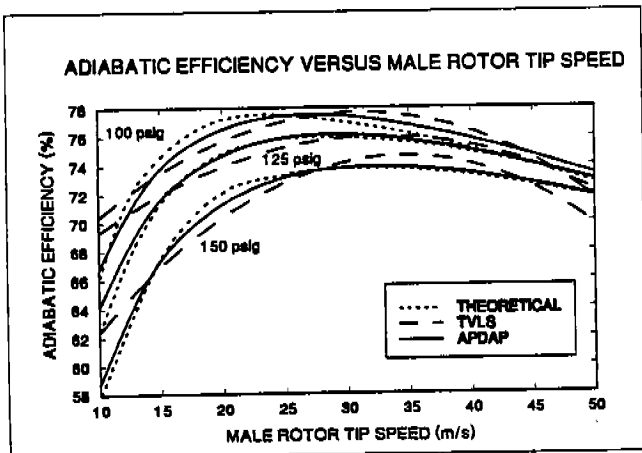
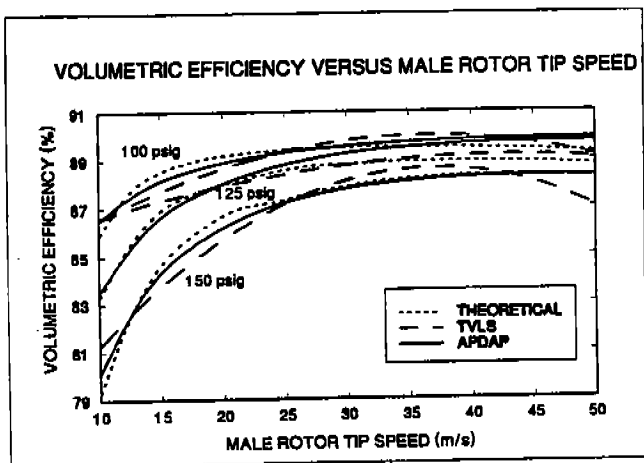
In general the results presented in Figures 1 and 2 show that while the APDAP model is only marginally better than the TVLS method in terms of how well it matches the actual data points, it results in a performance model that more closely resembles accepted performance characteristics for rotary screw compressors.

The most important test of a performance modeling program is how well it models actual test data. The results of APDAP on an actual test are shown in Figure 3. A TVLS model is included for comparison. This set of data is an example of how the APDAP model identifies questionable data points. In this particular case, the program identified the 10 m/s, 175 psig data point as being questionable. At the present time the program simply identifies these points. It is up to the engineer to resolve the issue. The questionable point in this illustration was removed from the model. The result was a model that correlated better to the remaining points and better reflects accepted knowledge of compressor performance based on experience and analysis.

## FUTURE WORK

Currently the APDAP model only includes shaft power, capacity, rotor speed, and discharge pressure. There are more variables which affect compressor performance and should be included. Oil flow rate affects performance but must be quantified independently of compressor size, and therefore should be considered carefully before adding it to the model. Correction of dynamometer data to standard inlet conditions is also a feature that may be added to the model. As mentioned earlier, as of now the dynamometer data read by APDAP has been corrected to a standard inlet pressure, but





**Figure 2. Comparison of APDAP and TVLS to Theoretical Model**

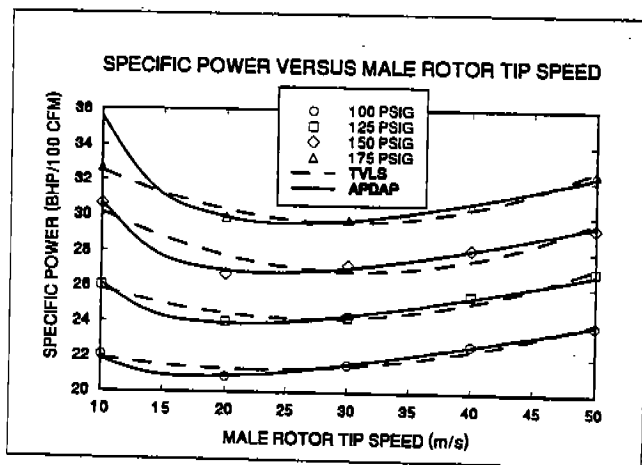
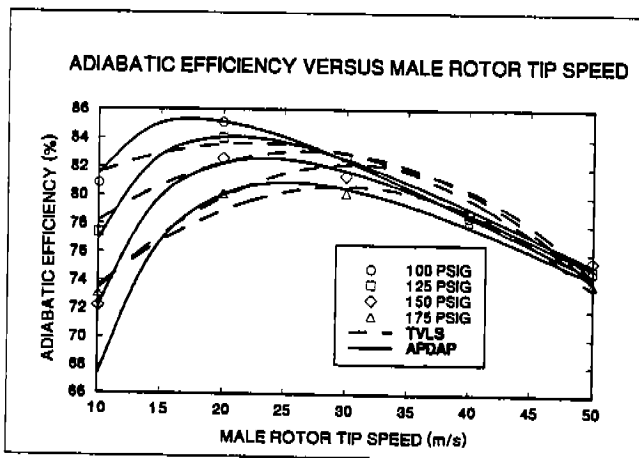
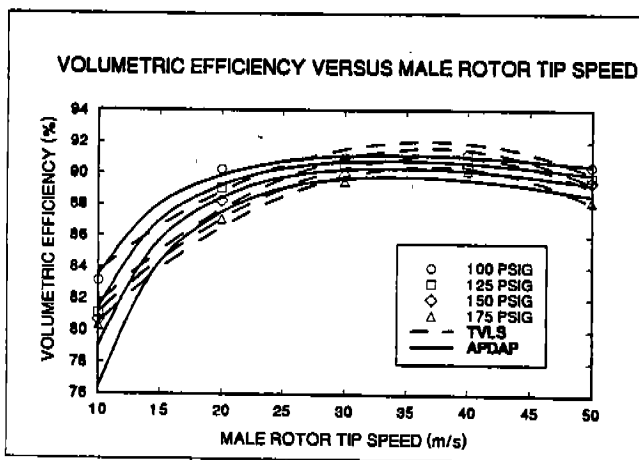


Figure 3. Comparison of APDAP and TVLS to Actual Test Data

there is room for improvement beyond this. The model could include correction terms for ambient, so that once performance had been determined at one set of conditions, it could easily be predicted at another. This same idea could also be applied to package losses, enabling the package engineer to accurately size coolers and other components in his effort to obtain peak performance. Finally, as the program is used to model a greater number of compressors and experience increases, it is hoped that some of this knowledge gained may be captured in the program, enabling it to grow into an expert system.

Other changes are being made on a system level rather than an analytical level. The program will be incorporated into the Engineering Technology Laboratory Integrated Test and Analysis System. This will enable the program to become a menu-driven module of the test network. The user will simply select the data set to be analyzed, and the appropriate files will be input to APDAP and the model calculated. A large portion of the module will be devoted to post-processing the model results. Data files and plots will be generated from the model as requested by the user. Eventually, the capability to produce two- and three-dimensional performance maps will be available. The idea is to allow the test, development, and package engineers to have performance curves and maps available minutes after completing a compressor test.

## CONCLUSIONS

Due to developments in the compressor industry, particularly emphasis on computer aided engineering, shorter product development cycles, and improved quality, current methods for evaluation of performance test results are no longer adequate. The Aired Performance Data Analysis Program (APDAP) was developed in response to deficiencies of current methods. This program has the following benefits.

1. The APDAP performance model more accurately represents the actual rotary screw compressor performance.
2. Questionable data points (data points that result in the model being contrary to accepted compressor performance characteristics) are identified so that the effect of normally encountered test error is minimized.
3. Because APDAP produces a single mathematical data base, the performance results are repeatable. Interpolation and interpretation of the data are not required to predict performance at any set of operating conditions.

The application of this analytical tool will result in timely, accurate data made available to the engineer in the accomplishment of technical evaluation of rotary screw compressor performance.

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